Tuesday, April 12, 2016
8:46 AM

At long last, after months of tedious work, I am giving up. From here on out we will adopt the usual metric convention $\eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$ used in particle physics. A big reason for this is that it is <u>the</u> convention used in particle physics contexts and the alternative is typically reserved for more formal QFT books.

With that in mind we should recall the changes this forces on us:

$$V^\mu = \begin{pmatrix} V^0 \\ V^1 \\ V^2 \\ V^3 \end{pmatrix} \Rightarrow V_\mu = (V^0, -V^1, -V^2, -V^3) \Rightarrow V^\mu V_\mu = V^{0\,2} - V^{1\,2} - V^{2\,2} - V^{3\,2}$$

$$\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2\phi^2 \qquad \Rightarrow \qquad \partial_\mu\partial^\mu\phi + \left(\frac{mc}{\hbar}\right)^2\phi = 0$$

$$\mathcal{L}_{Dirac} = \hbar c\bar{\psi}\gamma^\mu\partial_\mu\psi - mc^2\bar{\psi}\psi \qquad \Rightarrow \qquad i\gamma^\mu\partial_\mu\psi - \frac{mc}{\hbar}\psi = 0 \qquad \bar{\psi} \equiv \psi^\dagger\gamma^0, \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\mathcal{L}_{Proca} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} + \frac{1}{8\pi}\left(\frac{mc}{\hbar}\right)^2 A_\mu A^\mu \qquad \Rightarrow \qquad \partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) + \left(\frac{mc}{\hbar}\right)^2 A^\nu = 0$$

Monday, March 05, 2012
8:28 AM

What would we like to calculate (and compare to experiment)?

Decays $A \to \begin{cases} B + C & \Gamma_1 \\ D + E & \Gamma_2 \\ F + G + H & \Gamma_3 \\ \vdots & \end{cases}$

decay channels indexed by i

$A \to$ anything $\quad \Gamma_{tot} = \sum_i \Gamma_i$

Γ are "decay rates" = probability per unit time of decaying into ____.

Now: $dN = -\Gamma_{tot} N dt$
$\Downarrow$
$N(t) = N(0) e^{-\Gamma_{tot} t}$
$\Downarrow$
$\tau_{avg} = \frac{1}{\Gamma_{tot}}$ = "lifetime"

We will focus on calculating $\underbrace{\{\Gamma_i\}}_{\text{set of } \Gamma_i}$ from which we can get Γ_{tot}, τ_{avg}.
The "likelihood" of a particular set of decay products

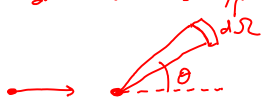
Collisions The "likelihood" of a particular collision event $A + B \to \underbrace{C + D}_{\text{outcome } i}$ is the scattering cross-section σ_i. The total or inclusive cross-section for $A + B$ is $\sigma_{tot} = \sum_i \sigma_i$.

We can contrast with a primitive scattering like firing an arrow at a target:
In this simple case the likelihood is really determined by the actual cross-sectional area of the target.

In particle physics, scattering is much more complicated:
- soft target (interaction w/ potential)
- depends on identity of arrow
- multiple ways to successfully "hit"
- velocity dependent
- in primitive case the final state is "hit" or "no hit", whereas in particles there are many possible outcomes.

We will focus on calculating $\underbrace{\{\sigma_i\}}_{\text{set of } \sigma_i}$.

Sometimes our view is limited to a small slice of solid angle $d\Omega$ (where detector sits).
So we might instead need $\frac{d\sigma}{d\Omega}$ which is typically only θ-dependent.



Our interest is in relativistic, quantum mechanical calculations of Γ_i, σ_i. This would really entail full QFT, but we will study and try to make sense of the result.

In both decays and scattering, the "likelihood" of an event is controlled by:

a) Kinematics (phase-space freedom), e.g. the larger the mass difference between in and out states, the more excess kinetic energy is liberated and this can be distributed in more ways in phase-space resulting in higher likelihood.

b) Dynamics (interactions), e.g. relative likelihoods governed by force strengths, intermediate states, etc.

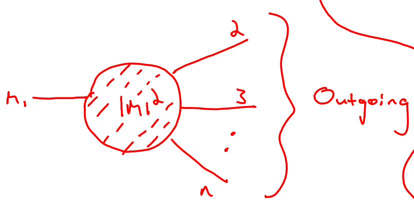
These two influences actually quasi-seperate in the final expressions for Γ_i and σ_i, so we can really handle them separately.

The kinematic contribution to Γ_i, σ_i is summed up in Fermi's Golden Rule (which works for any interaction):

Decay: $m_1^{rest} \rightarrow m_2 + m_3 + \dots + m_n$ (channel i)

All momenta p are 4-momenta!

$$\Gamma_i = \frac{S}{2\hbar m_1} \int |M|^2 (2\pi)^4 \delta^4(p_1 - p_2 - p_3 - \dots - p_n) \prod_{j=2}^{n} 2\pi \delta(p_j^2 - m_j^2 c^2)\, \Theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$



dynamics is contained in $|M|^2$

Enforces overall 4-momentum conservation

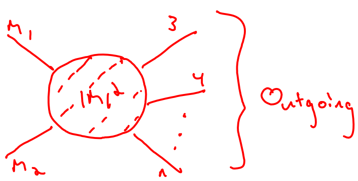
Forces outgoing particles to be real, i.e. satisfy mass-shell condition

Forces outgoing particles to have $E > 0$.

Adds up all possible kinematic configurations of outgoing.

Collisions: $m_1 + m_2 \rightarrow \underbrace{m_3 + m_4 + \dots + m_n}_{\text{channel } i}$

$$\sigma_i = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \int |M|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - \dots - p_n) \prod_{j=3}^{n} 2\pi \delta(p_j^2 - m_j^2 c^2)\, \Theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$



Note: $S = \frac{1}{S_1!} \frac{1}{S_2!} \cdots$ where S_l = # of identical out particles of type l

$|M|^2$ will carry all of the <u>dynamical</u> information.

The Golden Rule simply says that (dynamics aside) all kinematic configurations consistent with 4-momentum conservation, positive energy, and mass-shell conditions are equally likely. So the more of them there are, the higher the likelihood!!

Monday, March 19, 2012
7:22 PM

At this point we usually can't go further since $|M|^2$ will often depend on p_j and so we need it before integrating. But in a few special cases the kinematics is so tightly constrained that we can go a bit further.

First, we can <u>always</u> break up $d^4 p_j = dp_j^0 \, d\vec{p}_j$ and use $\delta(p_j^2 - m_j^2 c^2) = \delta(p_j^{0\,2} - \vec{p}_j^{\,2} - m_j^2 c^2)$ to perform the dp_j^0 integral using the properties that:

$$\delta(x^2 - k^2) = \frac{1}{2k}\left[\delta(x-k) + \delta(x+k)\right] \qquad k > 0$$

constant

Then:

$$\Gamma_i = \frac{S}{2\hbar m_1} \int |M|^2 (2\pi)^4 \delta^4(p_1 - p_2 - \cdots - p_n) \prod_{j=2}^{n} \frac{1}{2\sqrt{\vec{p}_j^{\,2} + m_j^2 c^2}} \frac{d^3\vec{p}_j}{(2\pi)^3}$$

$$\sigma_i = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \int |M|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - \cdots - p_n) \prod_{j=3}^{n} \frac{1}{2\sqrt{\vec{p}_j^{\,2} + m_j^2 c^2}} \frac{d^3\vec{p}_j}{(2\pi)^3}$$

Now for 2 cases that are so tightly constrained by the kinematics that we have enough δ-functions in FGR to let us evaluate all of the integrals w/out the functional form of $|M|^2$.

"2-body" decay $\quad 1 \to 2 + 3 \qquad \Gamma = \frac{S|\vec{p}|}{8\pi\hbar m_1^2 c} |M|^2$ where $|\vec{p}| = \frac{c}{2m_1}\sqrt{m_1^4 + m_2^4 + m_3^4 - 2m_1^2 m_2^2 - 2m_1^2 m_3^2 - 2m_2^2 m_3^2}$

at rest!

magnitude of momentum of <u>either</u> outgoing particle (same since $\vec{p}_{tot} = 0$)

If you fix m_1 and plot Γ as a function of m_2, m_3 you will find that it grows w/ increasing mass difference.

magnitude of momentum for either outgoing particle

"2-body" scattering in <u>CM frame</u> $\qquad \frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|M|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$

magnitude of momentum for either incoming particle

$1 + 2 \to 3 + 4$

$\vec{p}_{tot} = 0 = \vec{p}_{tot}$